



## Comparison of Students' Mathematical Creative Thinking Ability through Means-Ends Analysis and Discovery Learning Model

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**Abstract.** Creative thinking is one of mathematical abilities that students need to achieve. In fact, that ability is not yet fully in line with expectations. The purpose of this research is to analyze students' achievement and enhancement in mathematical creative thinking, particularly for those who acquired learning through means-ends analysis model and discovery learning model. The method used was quasi-experimental research design with two-group pretest-posttest design. This research was conducted to grade 8 students (13-14 years) in one of school located at Bandung Regencies and the number of samples involved was 68 students. The results showed that there were no differences in achievement and enhancement of mathematical creative thinking ability between students who acquired learning through means-ends analysis model and students who acquired learning through discovery learning model. The level of achievement in mathematical creative thinking abilities in both research classes were sufficient, while the level of the enhancement in mathematical creative thinking abilities in both research classes were moderate. Based on the results, means-ends analysis and discovery learning can be used as alternative learning models to develop students' creative thinking ability.

### 1. Introduction

Mathematical creative thinking ability is one of the abilities expected to be achieved by students through mathematics learning. This ability needs to be developed by students because it can build their positive attitude towards mathematics itself and can be used as a capital to make them become a more flexible person in facing situations in real life [1,2].

Mathematical creative thinking ability can be defined as a deep thinking process so that students are able to produce many answers, use different ideas or ways, and produce unusual answers [3,4]. This ability will train students to prepare themselves for the future challenges such as innovation and expansion of knowledge [2].



Some previous studies showed that mathematical creative thinking ability of junior high school students are relatively low. Research conducted by [5] showed that the criteria of students' achievement in the aspects of fluency and flexibility are less creative while the aspects of originality and elaboration are not creative. In addition, research conducted by [6] showed that the aspects of fluency and flexibility can only be achieved by students with high and moderate abilities, while the originality aspect can only be achieved by high-ability students. This indicates that mathematical creative thinking ability has not been achieved well by whole students.

The low of students' creative thinking ability can be influenced by several factors. One of them is influenced by classroom learning activities. Grégoire suggests that mathematical creative thinking ability are strongly influenced by how teacher build appropriate learning environments [7]. Based on this opinion, one way that can be taken to develop students' ability to think creatively is by choosing a learning model that can support students' creative thinking processes.

Learning activities that need to be developed to fulfill students' needs are learning environment that emphasizes how students learn, not on how teachers explain. McGregor suggests some activities that can support students' creative thinking in class, such as by providing more open learning opportunities or giving opportunities for problem solving activities, giving more open questions, giving students more time to develop the idea, and encouraging initial idea exploration activities [8].

Based on these opinions, mathematical creative thinking ability can be supported through learning that encourages students to carry out various investigations, so that students can obtain their own solutions to the problems given. One learning model that provides learning opportunities is means-ends analysis learning model. Simon & Newell suggest that means-ends analysis is a way to solve problems through a heuristic search system that is done by 1) identifying differences between current statements and expected goals; 2) analyzing the strategies needed to achieve problem solving; 3) applying the most likely strategy to achieve the expected goals [9]. This step is carried out by students by compiling sub- goals that are simpler than the problems given.

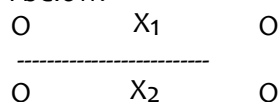


In addition to means-ends analysis, another learning model that provide opportunities for students to conduct various investigations, so they can find their own solutions to the problems given is discovery learning model. Discovery learning is a mental process carried out by students to assimilate concepts that consist of observing, grouping, submitting hypotheses, explaining, measuring, and inferring [10]. This opinion states that through discovery learning, students will acquire knowledge that they have not known through their own findings.

Some of the results of previous studies have shown that means-ends analysis and discovery learning models can enhance mathematical creative thinking ability, but there has been no research that shows differences in achievement and enhancement of mathematical creative thinking ability of both learning models. Therefore, this study aims to determine differences in achievement and enhancement of mathematical creative thinking abilities of students who acquired learning through means-ends analysis models and students who acquired learning through discovery learning models.

**2. Method**

The method used in this study was quasi-experimental method. This is based on the purpose and the selection of research subjects. The research design used was a two-group pretest-posttest design and can be described in Figure 1 below:



Annotation:

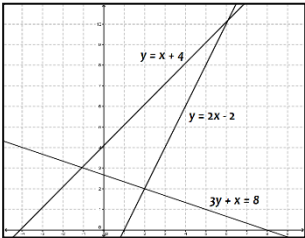
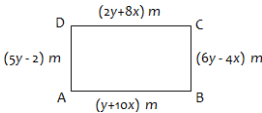
- O : Pretest, Posttest
- X<sub>1</sub> : Means-ends analysis learning model (MEA)
- X<sub>2</sub> : Discovery learning model
- : Subjects are not randomly grouped

**Figure 1.** Research Design

The population in this study were all grade eight students (13-14 years) in one of junior high schools located at Bandung Regency. Sampling was carried out by purposive sampling technique that was chosen through the consideration of grade eight mathematics teacher. The selection of these two classes was reviewed based on the characteristics and abilities of students. The number of samples involved was 68 students (34 students in *means-ends analysis* class and 34 students in discovery class).

Test instrument used were five questions related to System of Linear Equation in Two Variable that arranged based on aspects and indicators of mathematical creative thinking ability. The five aspects measured were fluency, flexibility, originality, elaboration, and sensitivity. The test instrument were given in the form of pretest and posttest. Table 1 shows instruments tested to students.

**Table 1.** Question Items on Mathematical Creative Thinking Ability Test

No	Creative Thinking Aspect	Question Items
1	Fluency	<p>Look at the following graph!</p>  <p>Write down at least three systems of linear equations in two variables shown in the graph, then write down the set completion!</p>
2	Flexibility	<p>Solve the following system of linear equation in two variables in various ways! (at least two ways)</p> $\begin{cases} 4x - y = 5 \\ x - 3y = -7 \end{cases}$
3	Elaboration	<p>Yoga and Rizal are 42 years old. If Yoga is 8 years older than Rizal, what is the age of Yoga and Rizal? Write down the steps to solve the problem in detail!</p>
4	Originality	<p>The picture below is a sketch of a rectangular garden plot owned by Mr Deni.</p> 



If Mr. Deni wants to build a fence around his garden, how meters is the length of the fence needed by Mr. Deni? Write down your way to get the length of the fence needed by Pak Deni!

- 5      Sensitivity      In mathematics learning, a teacher writes two line equations on the board as follows:

$$g : 3x - 5 = 15$$

$$: 6x - 10y = 12$$

After the teacher finished writing the equation, a student named Ahmad raised his hand and then said “The line  $g$  and line  $l$  intersect at a certain point, so that the SPLDV has exactly one solution”  
Do you think Ahmad's statement is correct? Give the reason!

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The implementation of this research is divided into three parts, that are pretest, implementation of learning model, and posttest. Students were asked to solve pretest for  $2 \times 40$  minutes. Then, every student in class of MEA and discovery acquired learning about System of Linear Equation in Two Variable for five meetings, with a duration of each meeting is  $2 \times 40$  minutes. Students were divided heterogeneously into small groups of four to five students to solve the problem given in students' worksheet. Finally, students were asked to solve posttest for  $2 \times 40$  minutes.

The processed data consists score of pretest, posttest, and N-Gain. First, the three data analyzed descriptively to find out the general data description. Then, hypothesis testing for each data was carried out. All statistical tests were calculated with IBM SPSS Statistics 20 software. Furthermore, the criteria for achieving and enhancing creative thinking abilities were used to determine the extent of students' achievement and enhancement in mathematical creative thinking abilities.



### 3. Result and Discussion

Data obtained from this research were score of pretest and posttest. Then, from the two score data, can be obtained the N-Gain value. Table 2 shows descriptive statistic of mathematical creative thinking abilities of students who obtained learning through means-ends analysis model and students who obtained learning through discovery learning model. Then, Table 3 shows the result of hypothesis testing.

**Table 2.** Descriptive Statistics of Pretest, Posttest, and N-Gain Score

Class	Pretest			Posttest			N-Gain		
	<i>n</i>	$\bar{x}$	<i>s</i>	<i>n</i>	$\bar{x}$	<i>s</i>	<i>n</i>	$\bar{x}$	<i>s</i>
MEA	34	4,24	1,67	34	10,94	3,33	34	0,43	0,20
Discovery	34	3,21	1,90	34	10,00	4,35	34	0,42	0,23
Ideal Score Maximum (ISM) = 20,00							ISM = 1,00		

**Table 3.** The Result of Hypothesis Testing

Score	Statistics			Annotation	Conclusion
	Mann-Whitney U	Z	Asymp. Sig. (2-tailed)		
Pretest	431,500	-1,830	0,067	Ho accepted	There is no difference in the initial ability of mathematical creative thinking between students in MEA classes and students in discovery classes.
Posttest	471,500	-1,312	0,190	Ho accepted	There is no difference in the achievement of mathematical creative thinking abilities between students in MEA classes and students in discovery classes.
N-Gain	531,000	-0,577	0,564	Ho accepted	There is no difference in the enhancement in mathematical creative thinking abilities between students in MEA classes and students in discovery classes.

Based on the results of hypothesis testing, it was found that there were no differences in



achievement and enhancement of mathematical creative thinking abilities between students in MEA class and students in discovery class. One possible reason is because both MEA and discovery support the creative thinking ability. Then, if viewed from every aspect of creative thinking abilities, it was found that the highest achievement and enhancement in MEA class is on the fluency aspect, while in discovery class is on the elaboration aspect. (See in Table 4)

In the MEA learning model, learning activities are emphasized on heuristic based problem solving. So that, in that learning activities, students are not only given a problem, but also equipped with a road map in the form of questions or commands at each step of problem solving [11]. Problem solving is one of activities that can support mathematical creative thinking abilities. Grégoire suggests that in problem solving, students need to be given the opportunity to find various answers to get the right solution [7]. The activity of trying various alternative solutions to the problem can supports students' mathematical creative thinking ability, especially on fluency aspects.

In discovery learning model, learning activities are emphasized on how students gain knowledge they have not yet known through their own findings. This learning activity encourages students to explore. McGregor suggested that one activity that can support students' creative thinking is to encourage exploration of the initial idea [8]. Based on this, before finding an answer, students need to do various experiments. This activity can support students' creative thinking ability, especially in elaboration aspects. In addition, generalization, as one of step in the implementation of discovery learning, train students to develop an idea. So, generalization can support students to think in detail.

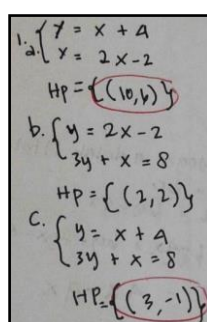
**Table 4.** Achievement and Enhancement Percentage

Class	Achievement Percentage				
	Fluency	Flexibility	Elaboration	Originality	Sensitivity
MEA	79	43	68	46	37
Discovery	54	58	70	30	38

Class	Enhancement Percentage				
	Fluency	Flexibility	Elaboration	Originality	Sensitivity
MEA	65	39	29	46	36
Discovery	45	54	56	21	30

The achievement category of mathematical creative thinking abilities of the two research classes is sufficient, while the category of enhancement mathematical creative thinking ability of the two research classes is moderate. This category has not reached the good and high category because there are some student still had difficulties. It can be seen from the mistakes made by students both in MEA class and discovery class. Difficulties experienced by students are caused by their unfamiliarity in working on questions that test creative thinking ability. Here are some examples of mistakes made by students both in MEA classes and discovery classes.



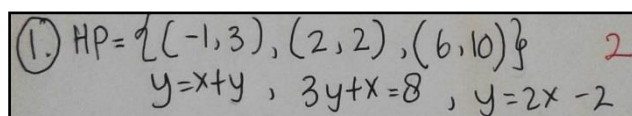
a.  $\begin{cases} y = x + 4 \\ x = 2x - 2 \end{cases}$   
 $HP = \{(10, 6)\}$

b.  $\begin{cases} y = 2x - 2 \\ 3y + x = 8 \end{cases}$   
 $HP = \{(2, 2)\}$

c.  $\begin{cases} y = x + 4 \\ 3y + x = 8 \end{cases}$   
 $HP = \{(3, -1)\}$

**Figure 2.** Student's Errors in Problems Number One in MEA Class

The mistake made by the student in Figure 2 is wrong in writing the coordinate point rules. The coordinate point should be written  $(x, y)$ , but students write it as  $(y, x)$ . The error of writing the coordinate rule caused the student incorrect to determine the set completion of SPLDV shown in the graph. The mistake made by students in Figure 3 is wrong in understanding the definition of SPLDV to answer the problem. Students write the equation  $y = x + 4$  has a set completion  $(1, -3)$ , the equation  $3y + x = 8$  has a set completion  $(2, 2)$  and for the equation  $y = 2x - 2$  has a set completion  $(6, 10)$ . This means that student only write one equation for a set completion, whereas a SPLDV has at least two equations. From the two examples of students' answers, the fluency aspect of students has just appeared not perfect.



① HP =  $\{(-1, 3), (2, 2), (6, 10)\}$  2  
 $y = x + 4, 3y + x = 8, y = 2x - 2$

**Figure 3.** Student's Errors in Problems Number One in Discovery Class

The types of mistakes made by students in these two classes tend to be the same. It is also causes no differences in achievement and enhancement in mathematical creative thinking abilities of students who acquired learning through means-ends analysis and discovery models.

#### 4. Conclusion

Based on the result and discussion, there are some conclusions are obtained: (1) The achievement of mathematical creative thinking abilities of students who acquired learning through means-ends analysis model is not significantly different from students who acquired learning through discovery models. The achievement of mathematical creative thinking ability in both research classes is in the sufficient level. The highest achievement for mathematical creative thinking ability aspects in MEA class is fluency, while in discovery class is elaboration; (2) The enhancement of mathematical creative thinking ability of students who acquired learning through means-ends analysis model are not significantly different from students who acquired learning through discovery. The enhancement of mathematical creative thinking in both classes is in the medium level. The highest enhancement for mathematical creative thinking ability aspects in MEA class is fluency, while in discovery class is elaboration.



## 5. Acknowledgements

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