



Student's Mathematical Argumentation in Solving Closed Problem

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Abstract. Argumentation as need for support student success have to integrated and developed in mathematics learning. In other to do so, students are able to show or explain the truth of a mathematical result with supporting facts and evidence. Mathematical learning tends have emphasized in the activities of the correct answers and the questions given have not provided an opportunity for students give mathematical argumentation. Therefore, students necessary habituation with mathematical argumentation through closed problem questions. Students' mathematical argumentation can identify and observe when students write their mathematical ideas. The aims of this study describe student's mathematical argumentation and types of argumentation when they solve closed problem. To investigated this issue, used descriptive method and carried out in written test four students junior high school. The results showed that students were abled to argue but have not been able to given mathematical argumentation comprehensive and correct. The conclusion of this study, students provide a clear and simple mathematical argumentation whose just contains objects and mathematical principles, verbal and pictorial argumentation are used by students in solving closed problem.

Keywords: argumentation, mathematical argumentation, types of argumentation, closed problem

1. Introduction

Reasoning is an important aspect in mathematics because it's always involves and inherent in every mathematical activity [1]. Reasoning as part of the five standard processes that are important for own by students [2] and compatible with purpose of mathematics learning in Indonesia that students are able to use deep reasoning on patterns, construct of proof and explain ideas or mathematical evidences [3]. Therefore, reasoning an important part to empowered mathematics learning which can see from student's argumentation ([4], [5], [6]). Argumentation is an important component in problem solving

[7] and as a foundation that can develop conceptual understanding, communication, and make logical connections between mathematical ideas ([8], [9], [10]) which are ultimately expect



become skills and life's proficiency will useful in the 21st century both in dialogue, negotiation, debate and deliberation [11].

The argumentation as a support for student success certainly needs have to integrated and developed in mathematics learning because mathematical argumentation is intend students are able to show or explain the truth of a mathematical outcome ([12], [13]). However, mathematics learning tends to emphasize in activity of finding the right answer and the question are not provided an opportunity for student give the argumentation so students still have difficulty in giving mathematical argumentation. This is supported by research studies who have conducted on mathematical argumentation of elementary, secondary and university.

The research has been done by [1] on class VIII SMP students on mathematical argumentation in which the results of their research students still had difficulties in agmentation. The same thing has been done by [14] regarding the mathematical argumentation of class VIII junior high school students where the results of their research illustrate that students have provided argumentation but have not been able to demonstrate the argumentation without mathematical rules. In addition, student's of junior high school, mathematical argumentation studies also have been conducted on students [7] who use problem- based-learning and conventional learning which shows both learning approaches don't have a significant influence on students' mathematical argumentation abilities. The same research [15] which comprehensively analyzed the ability of students' mathematical argumentation where the results of their research achieved the mathematical argumentation abilities of student students in the sufficient category. Some of the studies above, show that mathematical argumentation need have to improved and serve as a basis for conducting further studies on mathematical argumentation for both high school students and students's university. Thus, there needs to be an effort that can be done by training, teaching and student's habituation with argumentation which will later make students able to explain a claim or statement with accompanied by logical reasons based on facts and supporting evidence.

A good argumentation must certain specified criteria. John & Blair (in [16]) states the criteria for assessing arguments that can be used are *relevance*, *sufficiency*, *acceptability* and *clarity and expliciteness* [16]. The four criteria mentioned above mean that a good argument must contain clear sentences and not ambiguity, relevance, and adequacy supported by explanations of theories, evidence or facts so that they can be accepted in the context of mathematical disciplines or other disciplines. The aspects of mathematical argumentation derived from the four criteria of argumentation are good *clarity and expleciteness*, *relevance*, *sufficiency*, *acceptability*. The skills aspect used is evaluation that is operationalized into indicators a) evaluating the truth of a statement with supporting facts or evidence,

b) tracing the truth of a statement involving mathematical principles in accordance with the data provided, relevant guarantees and acceptable claims in determine a decision. Furthermore anchoring

[17] is an activity that is used to identify whether an argument belongs to mathematical argumentation or not by looking at components that include objects (*objects*), transformation (*transformation*) and concepts (*concepts*). The three components are closely related to the object of mathematical studies where objects in anchoring activities have similarities with facts, concepts meaning the same as abstract ideas and transformations mean similarities with the use skills or procedures in mathematics. This shows, not all arguments given by students are mathematical



argumentation so that there needs to be certain criteria in determining a student's mathematical argumentation.

Argumentation does not come naturally but is obtained through practice and habituation so that argumentation needs to be integrated in mathematics learning in the classroom. The practice and habituation of students in argumentation can be done by choosing mathematical tasks that provide opportunities for students to provide mathematical argumentation. According to [18] which states the identification of tasks that are appropriate in exploring students' abilities is important in the learning process. The math task that can be chosen is a task that contains problem solving. In solving problems, students carry out activities to understand problems, analyze problems, connect ideas or mathematical facts, concepts and principles that are used so that eventually students are able to explain the problem solving process and then conclude the solution to the problem given.

Problem solving in mathematics is divided into two types, namely open problems and closed problems ([19]; [20]). The difference between them in the number of solutions and how to solve them. In daily learning mathematics activities, closed problems often used in daily and assessments in semester to find one correct answer using several methods. Therefore, when solve closed problem students will easily give their arguments. Closed problem questions are used in the word problem that contain questions that stimulate students to give their argumentation because of mathematics learning has not been focused on optimizing students' ability to argue, even though students have the potential to give their argumentation which are shown through various representations in verbal form, variables, images, graphs, tables, equations or combinations of everything which is then grouped into verbal arguments (*narrative*), algebraic, and pictorial [21] which help make it easier for students to arrange or give mathematical argumentation.

Based on the explanation above, the closed problem question becomes a problem that is usually done by students in daily learning in classroom and mathematical argumentation have an important role for students so that it becomes the basis for the authors to be studied further. In this study, we will describe how students' mathematical argumentation are seen based on the criteria of good mathematical argumentation and the types of argumentation used to make it easier for students to construct their mathematical argumentation. Thus, this study aims to describe students' mathematical argumentations and types of argumentation when students solve closed problem.

2. Method of Research

This qualitative research used descriptive method because that attempted to described the facts systematically and the exact characteristics of the subjects studied [22]. We used a written test. The written test consisted of two questions (see Table 1) which was adapted and modification of the journals and 7th grade mathematics book. The questions that are given contain about quadrilateral and square area. The number of participants in this study were four student's junior high school. Participants worked on the questions simultaneously for 30 minutes. After that, we analyzed the students' answers.



Table 1. Mathematical argumentation's problem

No	Indicator	Problems
1	evaluate truth of a statement with supporting facts or evidence	A statement is given: "The diagonal of a rectangle must be longer than the other side". Verify the statement with clear reasons!
2	trace truth of a statement involving mathematical principles in accordance with the given data, relevant warrant and acceptable claims in determining a decision	It is known that the area of a rectangle is 64 cm^2 with a ratio of length and width is 4: 1. If it's in a rectangle shaped square and its length ¹ of the length of a rectangle, then the area of the rectangle is 36 cm^2 . Is it true that the area of the square is 36 cm^2 ? Include an explanation and reasons that support your answer

3. Result and Discussion

In this section we will explained the mathematical argumentation of students when solving closed problems and types of argumentation used by students to make it easier for students gives mathematical argumentation

Table 2 types of student's tmathematical argumentation

Subject	Types of argumentation	
	First problem	Second problem
S1	pictorial and verbal	verbal
S2	pictorial and verbal	verbal
S3	pictorial and verbal	verbal
S4	verbal and pictorial	pictorial and verbal

Before discussing further about students' mathematical argumentation, it will be explained first about the types of student's argumentation in answering the two questions. In this case, the type of argumentation is interpreted as a representation that facilitates students in arranged their mathematical argumentation. Representations that students represent the mental development process in expressing mathematical ideas that are visualized in verbal form, variables, images, graphs, tables, equations or combinations of all of them [23].

Based on the table above (see Table 2), the type of argumentation that is widely used by students is picture and verbal both on the first and second questions. In the first question, three of the four students used a type of pictorial and verbal argumentation while the rest used verbal and image argumentation. While in the second question, three of four students use verbal argumentation and a combination of images and verbal. The results of the analysis that we did, pictorial argumentation was used because with images will make it easier for students to understand the questions which will then be used to find the correct answers. In addition, in the first question it is known to build "rectangles" where they already have basic knowledge of quadrilateral buildings so that they will tend to given pictorial argumentation. This is because

students have experienced and passed the visualization stage as a stage in recognizing geometric forms and mentioning the names of geometric shapes given [24]. Verbal arguments that are interpreted as sentence arguments are used by students because they are used in learning mathematics themselves and in everyday life. This is what makes it easier for students to arranged mathematical argumentation. According to [14] states verbal arguments emphasize the use of daily language in the form of sentences so as to facilitate students to argue. The analysis that we conducted on the subject who used a combination of two types of argumentation because the student is to strengthen the answer so that he can convinced himself of the answers he has found. This is reasonable because convincing yourself is part of the main activities in the argumentation [9].

The results of our findings from the four student answers to the first question showed that students did not understand the questions well. Because of the four student answers there is not one picture that described a "diagonal" rectangle so that students do not understood the diagonal of a rectangle. The mathematical argumentation given by students in the first question (see Figures 4 and 3) shows the existing object according to the condition of the given problem so that the mathematical argumentation only contains the nature of a rectangle. Other arguments shown by students in the first question (see Figure 1) contain the principle or formula of a rectangle. The other students' argumentation (see Figure 3) have stated that the diagonal side of the rectangle is longer than the other side but not accompanied by facts, principles, and mathematical procedures to show the truth of the argument given. Thus, basically students have been able to argue but students have not been able to gave mathematical argumentation correctly and systematically.

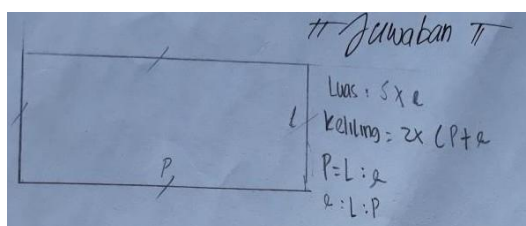


Figure 1. S1's answer first problem

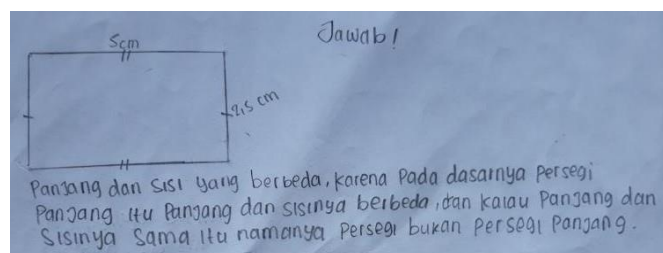


Figure 2. S2's answer first problem

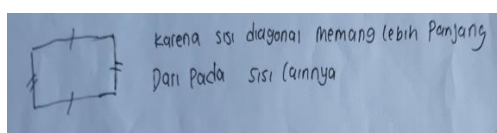


Figure 3. S3's answer first problem

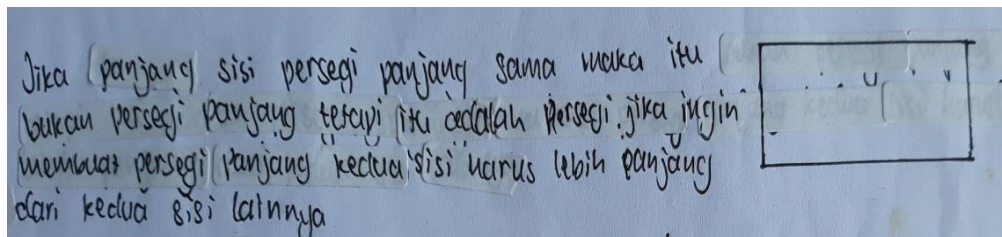
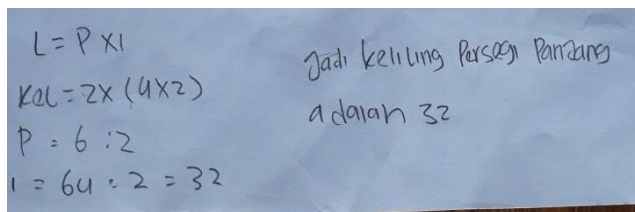


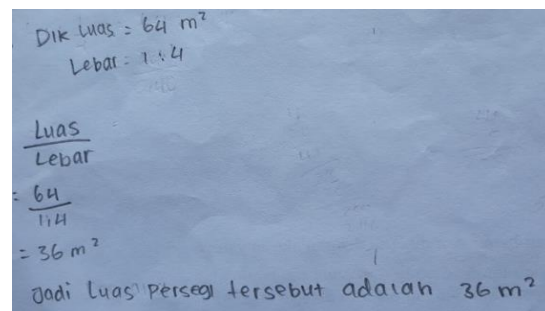
Figure 4. S4's answer first problem

Our findings in the students' answers to the second question, from the four students there were no correct answers. This is because students have not included the concept of comparison in finding the length of a rectangle. Of the four student answers, it seems that they have not understood the concept of comparison and the difficulty in solving problems related to the concept. Results of the study [25] on the *epistemological obstacles* concept of comparison in junior high school students who found a lack of students in applying conceptual understanding of ratios and proportions into contexts that had not been previously obtained by students. Based on students' answers, there are some students who assumed the length of the rectangle is 1.4 (see Figure 6). In addition, there are students who described a rectangle (see Figure 8) and the answers displayed do not contain the processed of students finding answers so we do not see the students' process in compiling mathematical argumentation (see Figure 7) and there are answers to students who have included principles and procedures mathematically but the answers given by students are not in accordance with the questions. Based on the things that have been explained, it appears that students do not understand what is meant by the problem and students appear to have not mastered the concept of comparison which then affects students in mathematical arguments. Identify the mathematical argumentation we made on the four student answers, two of the four answers students contain objects and mathematical principles but the answers given are not correct, while the rest contain mathematical objects but are not complete and the answers given are not correct. Thus, students have been able to provide arguments but the mathematical arguments are not correct, complete and systematic.



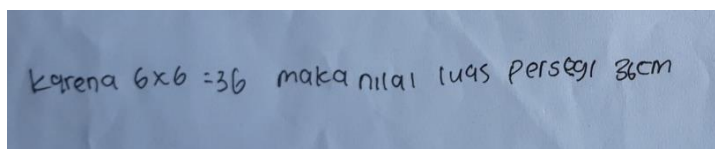
$L = P \times l$
 $Kel = 2 \times (4 \times 2)$
 $P = 6 : 2$
 $l = 64 : 2 = 32$
 Jadi keliling persegi panjang adalah 32

Figure 5. S1's answer second problem



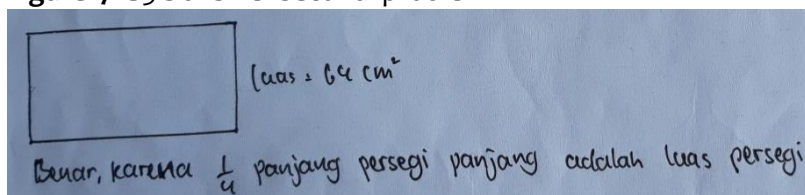
Dik Luas = 64 m^2
 Lebar = 1.4
 $\frac{\text{Luas}}{\text{Lebar}}$
 $= \frac{64}{1.4}$
 $= 36 \text{ m}^2$
 Jadi Luas persegi tersebut adalah 36 m^2

Figure 6. S2's answer second problem



Karena $6 \times 6 = 36$ maka nilai luas persegi 36 cm

Figure 7. S3's answer second problem



(Luas = 64 cm^2)
 Benar, karena $\frac{1}{4}$ panjang persegi panjang adalah luas persegi

Figure 8. S4's answer second problem

Based on good argumentation criteria, mathematical argumentation of the answers to the four subjects on the two questions is limited to clear and simple. The answers given explain the conditions that are in accordance with the problem not including concepts, rules, procedures and mathematical evidence so that the mathematical argumentation need to be improved again. Thus, the mathematical argumentation given by students will vary depending on the way students understand the questions, understanding and mastery of students 'mathematical concepts play a role in students' ability to provide mathematical argumentation.

4. Conclusion

Based on the description results that explain previously, the conclusions that we have obtained are that students able argumentation but have not been able to give mathematical argumentation correctly. The quality of mathematical argumentation is clear and simple and already contains objects, principles and procedures which applies in mathematics. The types of student's argumenatation use are verbal and pictorial argumenatation when solving closed problem.



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